Mr. Michael T. Davis Calculus	Section 1.5 Practice Quiz September 16, 2016

Name: Solution Key

Directions: Try each problem without your calculator, then use your calculator ONLY IF you feel you have truly tried everything possible without your calculator.

1. Determine if the function $f(x)=\ln x$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

   yes, for each y value in the range, there corresponds exactly one element in the domain.

2. Determine if the function $f(x)=|x|$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

   No, for most elements in the range, there corresponds two elements in the domain.

3. Determine if the function $f(x)=\frac{1}{x}$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

   Yes, for each element in the range, there corresponds exactly one element in the domain.

4. Determine if the function $f(x)=\sin x$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

   No, for each element in the range, there corresponds infinitely many elements in the domain.

5. Given the function $f(x)=2x-6$, determine the rule for the inverse function of $f(x)$.

   $f(x) = 2x-6$
   $y = 2x-6$
   $x = 2y-6$
   $2y = x+6$
   $y = \frac{1}{2}x+3$
   $f^{-1}(x) = \frac{1}{2}x+3$

6. The two functions $f(x)=\frac{3}{2}x-\frac{5}{2}$ and $g(x)=\frac{2}{3}x+\frac{5}{3}$ are inverses. Determine the value of the composite function $f(g(x))$. $f(g(x)) = f\left(\frac{2}{3}x+\frac{5}{3}\right) =$

7. A 1-to-1 function $y=f(x)$ is such that $f(14)=3$. Determine the value of $f^{-1}(3)$ $=14$

$= \frac{3}{2}\left(\frac{2}{3}x+\frac{5}{3}\right)-\frac{5}{2} = x+\frac{5}{2}-\frac{5}{2} = x \qquad f(g(x)) = x$

8. Solve the equation $\ln y = 3t - 2$ for y.

$$y = e^{3t-2} = e^{3t} \cdot e^{-2} = e^{3t} \cdot \frac{1}{e^2} = \frac{1}{e^2} \cdot e^{3t}$$

9. Solve the equation $5^t = 20$ for t.

$$\ln 5^t = \ln 20$$
$$t \ln 5 = \ln 20$$
$$t = \frac{\ln 20}{\ln 5} \approx 1.8614$$

10. Solve the equation $e^{2t} = 10$ for t.

$$\ln e^{2t} = \ln 10$$
$$2t = \ln 10$$
$$t = \frac{\ln 10}{2}$$
$$t = \frac{1}{2} \ln 10$$
$$t = \ln 10^{\frac{1}{2}}$$
$$t = \ln \sqrt{10}$$

11. Solve the equation $\log_m 81 = 4$ for m.

$$m^4 = 81$$
$$m = 3$$
$$m \neq -3$$

12. Solve the equation $\log_2 k = -4$ for k.

$$2^{-4} = K$$
$$\frac{1}{16} = K$$

13. Solve the equation $\log_2 t + \log_2 (t-6) = 4$ for t.

$$\log_2 (t^2 - 6t) = 4$$
$$2^4 = t^2 - 6t$$
$$0 = t^2 - 6t - 16$$
$$0 = (t-8)(t+2)$$
$$t = 8 \quad t \neq -2$$

14. Solve the inequality $\log x > 0$ for x.

$$x > 1$$

15. True or False: $\log_m (45) = \log_m 5 + \log_m 9$ $= \log_m (5 \cdot 9) = \log_m 45$ True

16. True or False: $\log_a\left(\frac{17}{11}\right)=\log_a 17-\log_a 11$ $= \log_a\left(\frac{17}{11}\right)$ True

17. True or False: $\log_b\left(x^5\right)=5\log_b(x)$ True

18. True or False: $\frac{\log_n 13}{\log_n 7}=\log_n 13-\log_n 7$ False
Not a property

19. True or False: $\ln\frac{w}{\sqrt{v}}=\ln w-\frac{1}{2}\ln v$ $= \ln w - \ln v^{\frac{1}{2}} = \ln w - \ln\sqrt{v} = \ln\left(\frac{w}{\sqrt{v}}\right)$
True

20. True or False: $\log_a x=\frac{\ln x}{\ln a}$ True
Change of Base Property

21. Evaluate or simplify the expression $8^{\log_8 15}$ $= 15$

22. Evaluate or simplify the expression $\log_9\left(9^y\right)$. $= y$

23. State the domain and range of the function $f(x)=\ln x$

D: $(0,\infty)$ R: $(-\infty,\infty)$

24. State the domain and range of the function $f(x)=\ln x+3$

D: $(0,\infty)$ R: $(-\infty,\infty)$

25. State the domain and range of the function $f(x)=\ln(x+3)$

D: $(-3,\infty)$ R: $(-\infty,\infty)$

26. State the domain and range of the function $f(t)=e^t$

D: $(-\infty,\infty)$ R: $(0,\infty)$

27. State the domain and range of the function $f(t)=e^t-3$

D: $(-\infty,\infty)$ R: $(-3,\infty)$

28. State the domain and range of the function $f(t)=e^{t-3}$

D: $(-\infty,\infty)$ R: $(0,\infty)$

29. The half-life of a certain radioactive substance is 12 days. There are 10 grams initially. Express the amount of substance remaining as a function of time t, where t is in days. In how many days will there be 2 grams remaining.

$A(t) = 10\left(\frac{1}{2}\right)^{\frac{t}{12}}$

$2 = 10\left(\frac{1}{2}\right)^{\frac{t}{12}}$

$\frac{1}{5} = \left(\frac{1}{2}\right)^{t/12}$

$\ln\left(\frac{1}{5}\right) = \ln\left(\frac{1}{2}\right)^{t/12}$

$\ln\left(\frac{1}{5}\right) = \frac{t}{12}\ln\left(\frac{1}{2}\right)$

$\frac{12\ln\left(\frac{1}{5}\right)}{\ln\left(\frac{1}{2}\right)} = t$

$t \approx 27.86$ days

$t \approx 27$ days & 21 hours

30. Determine how much time is required for a \$500 investment to double in value if the interest is earned at a rate of 5.75% compounded monthly.

$A(t) = 500\left(1+\frac{0.0575}{12}\right)^{12t}$ t = years

$1000 = 500\left(1+\frac{0.0575}{12}\right)^{12t}$

$2 = \left(1+\frac{0.0575}{12}\right)^{12t}$

$\ln 2 = 12t\ln\left(1+\frac{0.0575}{12}\right)$

$\frac{\ln 2}{12\ln\left(1+\frac{0.0575}{12}\right)} = t$

$t \approx 12.084$ years